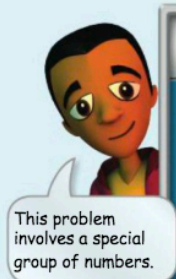
1-3 Real Numbers and the Number Line

Common Core State Standards

Prepares for N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational . . .

MP 1, MP 3, MP 6

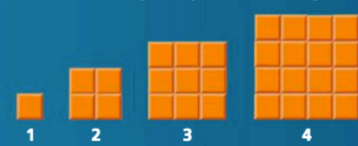
Objectives To classify, graph, and compare real numbers
To find and estimate square roots



MATHEMATICAL PRACTICES

Solve It! Getting Ready!

If the pattern continues, which will be the first figure to contain more than 200 square units? Explain your reasoning.



The diagrams in the Solve It model what happens when you multiply a number by itself to form a product. When you do this, the original number is called a *square root* of the product.

Lesson Vocabulary

- square root
- radicand
- radical
- perfect square
- set
- element of a set
- subset
- rational numbers
- natural numbers
- whole numbers
- integers
- irrational numbers
- real numbers
- inequality

take note

Key Concept Square Root

Algebra A number a is a **square root** of a number b if $a^2 = b$.

Example $7^2 = 49$, so 7 is a square root of 49.

Essential Understanding You can use the definition above to find the exact square roots of some nonnegative numbers. You can approximate the square roots of other nonnegative numbers.

The radical symbol $\sqrt{\ }$ indicates a nonnegative square root, also called a *principal square root*. The expression under the radical symbol is called the **radicand.**

radical symbol → $\sqrt{a}$ ← radicand

Together, the radical symbol and radicand form a **radical.** You will learn about negative square roots in Lesson 1-6.

...an you find a square root?
Find a number that you can multiply by itself to get a product that is equal to the radicand.

Problem 1 Simplifying Square Root Expressions

What is the simplified form of each expression?

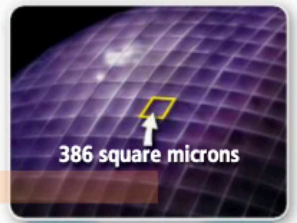
A $\sqrt{81} = 9$ $\quad 9^2 = 81$, so 9 is a square root of 81.

B $\sqrt{\frac{9}{16}} = \frac{3}{4}$ $\quad \left(\frac{3}{4}\right)^2 = \frac{9}{16}$, so $\frac{3}{4}$ is a square root of $\frac{9}{16}$.

Got It? **1.** What is the simplified form of each expression?

a. $\sqrt{64}$ **b.** $\sqrt{25}$ **c.** $\sqrt{\frac{1}{36}}$ **d.** $\sqrt{\frac{81}{121}}$

The square of an integer is called a **perfect square.** For example, 49 is a perfect square because $7^2 = 49$. When a radicand is not a perfect square, you can estimate the square root of the radicand.



Problem 2 Estimating a Square Root STEM

Biology **Lobster eyes are made of tiny square regions. Under a microscope, the surface of the eye looks like graph paper. A scientist measures the area of one of the squares to be 386 square microns. What is the approximate side length of the square to the nearest micron?**

...you get ...?
The square root of the area of a square is equal to its side length. So, find $\sqrt{386}$.

Method 1 Estimate $\sqrt{386}$ by finding the two closest perfect squares.

The perfect squares closest to 386 are 361 and 400.

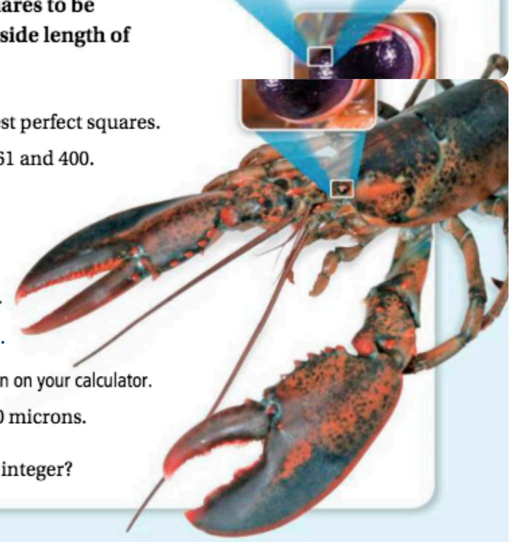
$19^2 = 361$
$\leftarrow$ 386
$20^2 = 400$

Since 386 is closer to 400, $\sqrt{386} \approx 20$, and the side length is about 20 microns.

Method 2 Estimate $\sqrt{386}$ using a calculator.

$\sqrt{386} \approx 19.6$ Use the square root function on your calculator.

The side length of the square is about 20 microns.



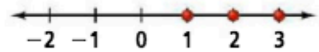
Got It? **2.** What is the value of $\sqrt{34}$ to the nearest integer?

Essential Understanding Numbers can be classified by their characteristics. Some types of numbers can be represented on the number line.

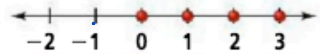
You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object is called an **element of the set.** A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

A **rational number** is any number that you can write in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A rational number in decimal form is either a terminating decimal such as 5.45 or a repeating decimal such as 0.41666 . . . , which you can write as $0.41\overline{6}$. Each graph below shows a subset of the rational numbers on a number line.

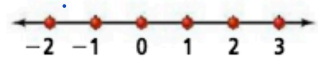
Natural numbers {1, 2, 3, . . .}



Whole numbers {0, 1, 2, 3, . . .}



Integers {. . . −2, −1, 0, 1, 2, 3, . . .}



An **irrational number** cannot be represented as the quotient of two integers. In decimal form, irrational numbers do not terminate or repeat. Here are some examples.

0.1010010001 . . . $\quad\quad \pi = 3.14159265\ldots$

Some square roots are rational numbers and some are irrational numbers. If a whole number is not a perfect square, its square root is irrational.

Rational $\quad \sqrt{4} = 2 \quad\quad \sqrt{25} = 5$

Irrational $\quad \sqrt{3} = 1.73205080\ldots \quad\quad \sqrt{10} = 3.16227766\ldots$

Rational numbers and irrational numbers form the set of **real numbers.**

Think

What clues can you use to classify real numbers?
Look for negative signs, fractions, decimals that do or do not terminate or repeat, and radicands ... not perfect

Problem 3 Classifying Real Numbers

To which subsets of the real numbers does each number belong?

A 15 natural numbers, whole numbers, integers, rational numbers

B −1.4583 rational numbers (since −1.4583 is a terminating decimal)

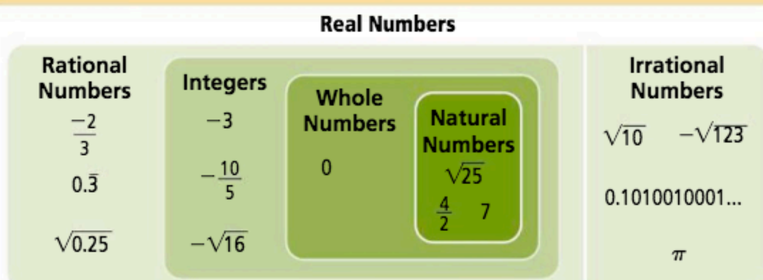
C $\sqrt{57}$ irrational numbers (since 57 is not a perfect square)

Got It? 3. To which subsets of the real numbers does each number belong?

a. $\sqrt{9}$ b. $\frac{3}{10}$ c. −0.45 d. $\sqrt{12}$

take note

Concept Summary Real Numbers



An **inequality** is a mathematical sentence that compares the values of two expressions using an inequality symbol. The symbols are

$<$, less than	$\leq$, less than or equal to
$>$, greater than	$\geq$, greater than or equal to

Plan

...you ...e numbers?
Write the numbers in the same form, such as decimal form.

Problem 4 Comparing Real Numbers

What is an inequality that compares the numbers $\sqrt{17}$ and $4\frac{1}{3}$?

$\sqrt{17} = 4.12310\ldots$ Write the square root as a decimal.

$4\frac{1}{3} = 4.\overline{3}$ Write the fraction as a decimal.

$\sqrt{17} < 4\frac{1}{3}$ Compare using an inequality symbol.

Got It? **4. a.** What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?

b. Reasoning In Problem 4, is there another inequality you can write that compares the two numbers? Explain.

You can graph and order all real numbers using a number line.

Problem 5 Graphing and Ordering Real Numbers

Multiple Choice What is the order of $\sqrt{4}$, 0.4, $-\frac{2}{3}$, $\sqrt{2}$, and -1.5 from least to greatest?

(A) $-\frac{2}{3}$, 0.4, -1.5, $\sqrt{2}$, $\sqrt{4}$

(B) -1.5, $\sqrt{2}$, 0.4, $\sqrt{4}$, $-\frac{2}{3}$

(C) -1.5, $-\frac{2}{3}$, 0.4, $\sqrt{2}$, $\sqrt{4}$

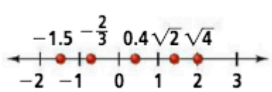
(D) $\sqrt{4}$, $\sqrt{2}$, 0.4, $-\frac{2}{3}$, -1.5

Know	Need	Plan
Five real numbers	Order of numbers from least to greatest	Graph the numbers on a number line.

Think

Why is it useful to rewrite numbers in decimal form?
It allows you to compare numbers whose values are close, like $\frac{1}{4}$ and 0.26.

First, write the numbers that are not in decimal form as decimals: $\sqrt{4} = 2$, $-\frac{2}{3} \approx -0.67$, and $\sqrt{2} \approx 1.41$. Then graph all five numbers on the number line to order the numbers, and read the graph from left to right.



From least to greatest, the numbers are -1.5, $-\frac{2}{3}$, 0.4, $\sqrt{2}$, and $\sqrt{4}$. The correct answer is C.

Got It? **5.** Graph 3.5, -2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?

Lesson Check

Do you know HOW?

Name the subset(s) of the real numbers to which each number belongs.

1. $\sqrt{11}$
2. -7
3. Order $\frac{47}{10}$, 4.1, -5, and $\sqrt{16}$ from least to greatest.
4. A square card has an area of 15 in.^{2}. What is the approximate side length of the card?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Vocabulary** What are the two subsets of the real numbers that form the set of real numbers?
6. **Vocabulary** Give an example of a rational number that is not an integer.

Reasoning **Tell whether each square root is *rational* or *irrational*. Explain.**

7. $\sqrt{100}$
8. $\sqrt{0.29}$

1-3 Real Numbers and the Number Line

Vocabulary

Review

1. Circle the numbers that are *perfect squares*.

1   12   16   20

100   121   200   289

Vocabulary Builder

square root (noun) **skwer root**

Definition: The **square root** of a number is a number that when multiplied by itself is equal to the given number.

Using Symbols: $\sqrt{16} = 4$

Using Words: The **square root** of 16 is 4. It means, "I multiply 4 by itself to get 16."

square root

$\sqrt{16} = 4$

because

$4^2 = 16$

Use Your Vocabulary

2. Use what you know about *perfect squares* and *square roots* to complete the table.

Number	Number Squared
1	1
2	4
3	
4	
5	
	36

Number	Number Squared
7	49
	64
	81
11	

Problem 1 Simplifying Square Root Expressions

Got It? **What is the simplified form of $\sqrt{64}$?**

3. Circle the equation that uses the positive square root of 64.

$16 \cdot 4 = 64$	$32 \cdot 2 = 64$	$8 \cdot 8 = 64$

4. The simplified form of $\sqrt{64}$ is ____.

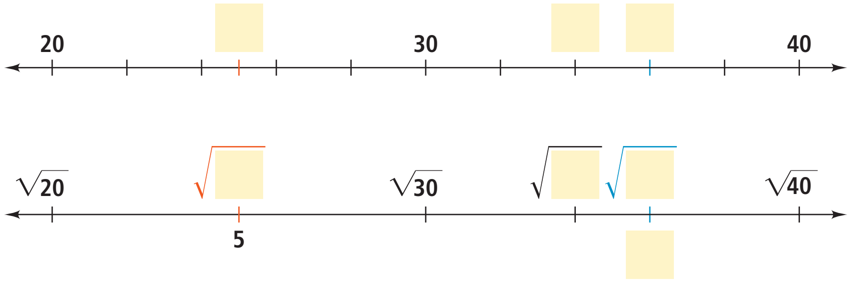
Problem 2 Estimating a Square Root

Got It? **What is the value of $\sqrt{34}$ to the nearest integer?**

5. Use the number lines below to find the perfect squares closest to 34.

Write 25, 34, and 36 in the correct positions on the number line.

Complete the number line with square roots.



6. Since 34 is closer to ____ than to ____,

$\sqrt{34}$ is closer to ____ than to ____.

So, the value of $\sqrt{34}$ to the nearest integer is ____.

You can classify numbers using *sets*. A **set** is a well-defined collection of objects. Each object in the set is called an **element** of the set. A **subset** of a set consists of elements from the given set. You can list the elements of a set within braces { }.

7. Complete the *sets* of numbers.

Natural numbers $\{1, \_\_, 3, \ldots\}$

Whole numbers $\{\_\_, 1, \_\_, 3, \ldots\}$

Integers $\{\ldots, -2, \_\_, 0, 1, \_\_, 3, \ldots\}$

A **rational number** is any number that you can write in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. A rational number in decimal form is either a terminating decimal such as 5.45 or a repeating decimal such as 0.333..., which you can write as $0.\overline{3}$.

8. Cross out the numbers that are NOT *rational numbers*.

π	$-\frac{7}{4}$	$\sqrt{5}$	$0.\overline{9}$	7.35

An **irrational number** cannot be represented as the quotient of two integers. In decimal form, irrational numbers do not terminate or repeat. Irrational numbers include π and $\sqrt{2}$.

Problem 3 Classifying Real Numbers

Got It? **To which subsets of the real numbers does each number belong?**

$\sqrt{9}$ $\quad\quad$ $\frac{3}{10}$ $\quad\quad$ -0.45 $\quad\quad$ $\sqrt{12}$

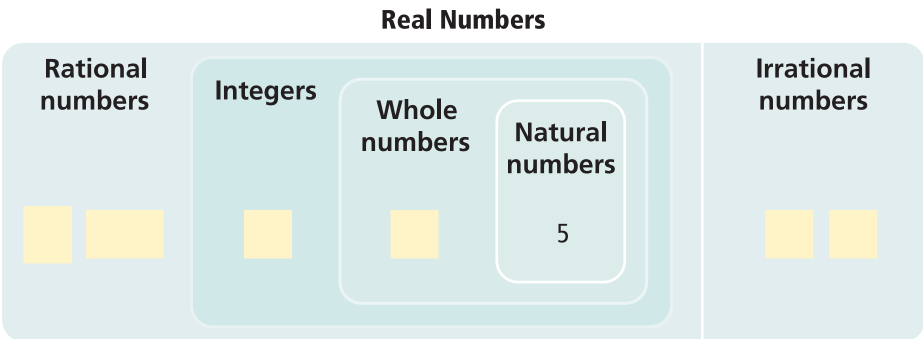
9. Is each number an element of the set? Place a ✔ if it is. Place an ✗ if it is not.

Number	Whole Numbers	Integers	Rational Numbers	Irrational Numbers
$\sqrt{9}$	✔	✔	✔	✗
$\frac{3}{10}$				
-0.45				
$\sqrt{12}$				

take note

Concept Summary Real Numbers

10. Write each of the numbers -7, -5.43, 0, $\frac{3}{7}$, π, and $\sqrt{7}$ in a box below. The number 5 has been placed for you.



Problem 4 Comparing Real Numbers

Got It? **What is an inequality that compares the numbers $\sqrt{129}$ and 11.52?**

11. What is the approximate value of $\sqrt{129}$ to the nearest hundredth?

12. Use $<$, $>$, or $=$ to complete the statement.

$\sqrt{129}$ ☐ 11.52

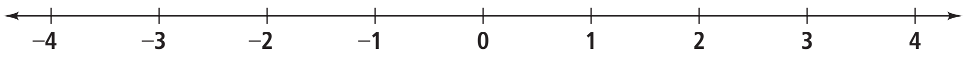
Problem 5 Graphing and Ordering Real Numbers

Got It? **Graph 3.5, −2.1, $\sqrt{9}$, $-\frac{7}{2}$, and $\sqrt{5}$ on a number line. What is the order of the numbers from least to greatest?**

13. Simplify the radicals and convert the fraction to a mixed number.

$\sqrt{9} =$ ☐ $-\frac{7}{2} =$ ☐ $\sqrt{5} \approx$ ☐

14. Now use the number line to graph the five original numbers. Be sure to label each point with the correct number.



15. Now list the five original numbers from *least* to *greatest*.

☐, ☐, ☐, ☐, ☐

Lesson Check • Do you UNDERSTAND?

Reasoning **Tell whether $\sqrt{100}$ and $\sqrt{0.29}$ are *rational* or *irrational*. Explain.**

16. First try to simplify the expression. If it does not simplify, put an ✗ in the box.

$\sqrt{100} =$ ☐ $\sqrt{0.29} =$ ☐

17. Tell whether each square root is *rational* or *irrational*. Explain your reasoning.

____

____

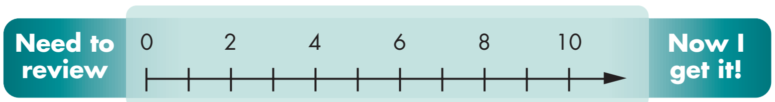
____

Math Success

Check off the vocabulary words that you understand.

☐ square root ☐ rational numbers ☐ irrational numbers ☐ real numbers

Rate how well you can *classify and order real numbers*.



Name____________________________________ Class_____________ Date_____________

1-3 Practice

Simplify each expression.

1. $\sqrt{4}$
2. $\sqrt{36}$
3. $\sqrt{25}$
4. $\sqrt{81}$
5. $\sqrt{121}$
6. $\sqrt{169}$
7. $\sqrt{625}$
8. $\sqrt{225}$
9. $\sqrt{\frac{64}{9}}$
10. $\sqrt{\frac{25}{81}}$
11. $\sqrt{\frac{225}{169}}$
12. $\sqrt{\frac{1}{625}}$
13. $\sqrt{0.64}$
14. $\sqrt{0.81}$
15. $\sqrt{6.25}$

Estimate the square root. Round to the nearest integer.

16. $\sqrt{10}$
17. $\sqrt{15}$
18. $\sqrt{38}$
19. $\sqrt{50}$
20. $\sqrt{16.8}$
21. $\sqrt{37.5}$
22. $\sqrt{67.5}$
23. $\sqrt{81.49}$
24. $\sqrt{121.86}$

Find the approximate side length of each square figure to the nearest whole unit.

25. a rug with an area of 64 ft^2
26. an exercise mat that is 6.25 m^2
27. a plate that is 49 cm^2

Name________________________________ Class______________ Date______________

1-3 Practice (continued)

Name the subset(s) of the real numbers to which each number belongs.

28. $\frac{12}{18}$ **29.** -5 **30.** π **31.** $\sqrt{2}$

32. 5564 **33.** $\sqrt{13}$ **34.** $-\frac{4}{3}$ **35.** $\sqrt{61}$

Compare the numbers in each exercise using an inequality symbol.

36. $\sqrt{25}, \sqrt{64}$ **37.** $\frac{4}{5}, \sqrt{1.3}$ **38.** $\pi, \frac{19}{6}$

39. $\sqrt{81}, -\sqrt{121}$ **40.** $\frac{27}{17}$, 1.7781356 **41.** $-\frac{14}{15}, \sqrt{0.8711}$

Order the numbers from least to greatest.

42. $1.875, \sqrt{64}, -\sqrt{121}$ **43.** $\sqrt{0.8711}, \frac{4}{5}, \sqrt{1.3}$ **44.** $8.775, \sqrt{67.4698}, \frac{64.56}{8.477}$

45. $-\frac{14}{15}, 5.587, \sqrt{81}$ **46.** $\frac{100}{22}, \sqrt{25}, \frac{27}{17}$ **47.** $\pi, \sqrt{10.5625}, -\frac{15}{5.8}$

48. Marsha, Josh, and Tyler are comparing how fast they can type. Marsha types 125 words in 7.5 minutes. Josh types 65 words in 3 minutes. Tyler types 400 words in 28 minutes. Order the students according to who can type the fastest.